

# Modifikacija modela hidroelektricne centrale preko izmerenih odziva realne centrale

## Modification of Nonlinear Hydro Power Plant Models Using Real Plant Measurements

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**Abstract** - Simulation models of Hydro Power Plant (HPP) are complex and highly nonlinear control system, composed of tunnel, penstock, surge tank, hydro turbine and hydroelectrical governor models. Hydro Power Plant (HPP) models recommended by the IEEE working group is revised and is used for simulation of the transient and static response of the system of hydro turbine and auxiliary equipment. Verification was made using measurements of static and transient responses from real HPP. Modification of simulation model is made after comparison of the simulated response of the HPP simulation model and real response of the HPP with PID controller. Modification and adjustment of the two parameters of the model is made to adjust the static response of the model without affecting transient response and with consideration of smallest frequency and power error.

**Index Terms** - Hydro Power Plant, Hydro Turbine, Nonlinear Model, Verification

**Apstrakt** - Simulacijski modeli hidroelektrane (HE) su kompleksni i visoko nelinearni upravljački modeli i sastoje se od modela hidrauličnog tunela, pritiskog cevovoda, vodostana, hidraulične turbine i upravljačkog sistema. Modeli hidroelektrane (HE) preporučeni od strane IEEE radne grupe su revidirani i koriste se za simulaciju dinamičkog prelaznog i stacionarnog odziva hidro turbine sa pomocnom opremom. Verifikacija modela je napravljena pomoću merenja stacionarnog i prelaznog odziva realne HE. Modifikacija simulacijskog modela je napravljena preko uporedbe simuliranog odziva i realno merenih odziva HE sa upravljačkim PID kontrolerom. Modifikacija i podesavanje dva parametra simulacijskog modela je učinjeno preko podesavanja stacionarnog odziva modela i merenih odziva, bez da se utiče na prelazne rezime odziva modela, uzimajući u obzir najmanju gresku frekvencije i snage.

**Ključne reči** - hidroelektrana, hidraulična turbina, nelinearni model, verifikacija

### I. INTRODUCTION

The requests of mathematical model of hydraulic plants including hydraulic turbine, penstock, unrestricted head and tail race, large or no surge tank and governor of hydraulic turbine are, to assure large transient stability program simulation, isolated system operation, system restoration after brake up, load rejection, load acceptance, water hammer dynamics and optimal speed control. In this paper are used for simulation both linear and non-linear mathematical models of hydraulic plants recommended by IEEE [1] and their modifications made with data from hydraulic turbine model acceptance measurements [2], which minimise the model generality, but maximise the similarity of model with real hydraulic turbine, on which model acceptance measurements is made. Non-linear models are required where speed and power changes are large, such as in islanding, load rejection and system restoration studies although there are great difficulties designing good governor of hydraulic turbines, because the hydraulic turbine is highly non-linear device which characteristics vary significantly with the unpredictable load on the unit. Such nonlinearities make the governor design a nontrivial task because governors designed for one operating condition may not work at all under other conditions. Because of that robust control law is designed, allowing the system hydraulic turbine-governor to work satisfactory at all working conditions, not only around working point. The significance of robust control design is to show how to overcome some of limitations of conventional governor design methods. Basic elements of a hydraulic turbine within the power system environment are shown on the block diagram of figure 1.

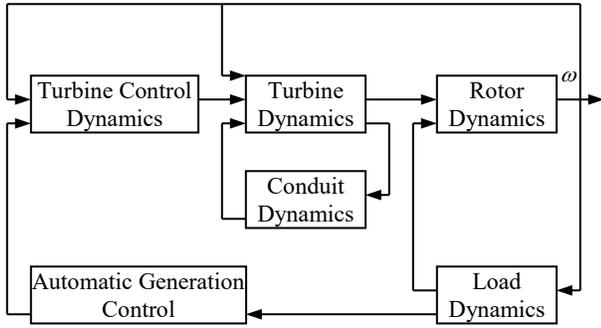


Figure 1. Block diagram of hydro prime mover and control

## II. MATHEMATICAL MODEL OF HYDRAULIC TURBINE

Mathematical models of hydraulic turbine with penstock and without surge tank, modelled assuming incompressible fluid, recommended by IEEE [1], [4] are presented. Modelling of linear and non-linear mathematical model of hydraulic turbine is presented in following subsections.

### A. Non-linear model of hydraulic turbine assuming incompressible flow

From the laws of momentum, the rate of change of flow in the conduit is:

$$\rho L \frac{dQ}{dt} = (H_s - H_l - H) A \rho g \quad (1)$$

where  $Q$  is the volumetric flow rate,  $L$  is the penstock length and  $\rho$  is the mass density of water. The net force on the water can be obtained by considering the pressure head at either end of the conduit. On entry to the penstock the force on the water is simply proportional to the static head  $H_s$  whilst at the wicket gate it is proportional to the head  $H$  across the turbine. Due to friction effects in the conduit there is also a friction force on the water represented by the head loss  $H_l$  so that the net force on the water is represented with the right side of the equation 1. It is usual to normalise equation 1 to a convenient base. Although this base system is arbitrary the base head  $h_{base}$  is taken as the static head above the turbine, which is equal to  $H_s$ , whilst the base flow rate  $q_{base}$  is taken as the flow rate through the turbine with the gates fully open and the head at the turbine equal to  $h_{base}$ . Dividing both sides of equation 1 with product  $h_{base} q_{base}$  gives:

$$\frac{dq}{dt} = \frac{1}{T_w} (1 - h_l - h) \quad (2)$$

where  $q = Q/q_{base}$  and  $h = H/h_{base}$  are the normalised flow rates and pressure heads respectively and  $T_w = \frac{Lq_{base}}{Agh_{base}}$  is the

water starting time. Theoretically  $T_w$  is defined as the time taken for the flow rate in the penstock to change by a value equal to  $q_{base}$  when the head term in the brackets changes by a value equal to  $h_{base}$ . The head loss  $h_l$  is proportional to the flow rate squared and depends on the conduit dimensions and friction factor, but here it suffices to assume that  $h_l = k_f q^2$  and can often be neglected. In turbine model, must be modelled hydraulic characteristics and mechanical power output. Firstly, the pressure head across the turbine is related to the flow rate to assuming the turbine can be represented by the normalized (per unit) valve characteristics:

$$q = c\sqrt{h} \quad (3)$$

where  $c$  is the gate position between 0 and 1.

The power developed by the turbine is proportional to the product of the flow rate and the head and depends on the efficiency, which is taken into account with subtraction of the no load flow  $q_{nl}$  term from the actual flow.

$$P_m = A_t h (q - q_{nl}) = \frac{\text{turbine power (MW)}}{\text{generator MVA rating}} \quad (4)$$

where the factor  $A_t$  is introduced to account the difference between turbine power and generator bases as shown in equation 4 and its values can be obtained by considering the operation of the turbine at rated load. It must be taken into account the damping effect that is dependent on gate opening, so that at any load condition the turbine power can be expressed by:

$$P_m = A_t h (q - q_{nl}) - Dc\Delta\omega \quad (5)$$

where  $D$  is the damping coefficient. Equations 2, 3 and 5 constitute the non-linear turbine model shown on the figure 2 where the wicket gate position is the control variable.

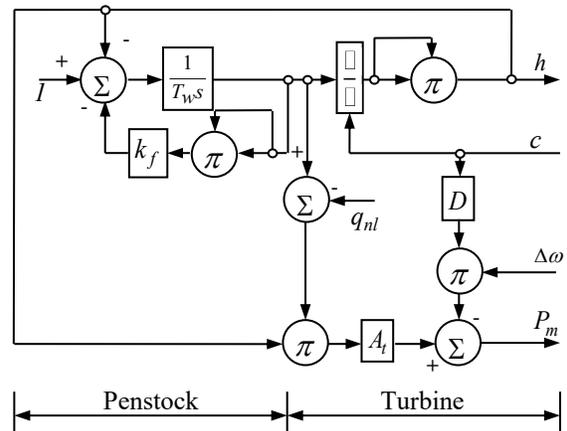


Figure 2. Non-linear model of hydraulic turbine - incompressible flow

### B. Linear model of hydraulic turbine assuming incompressible flow

Linearised mathematical model of hydraulic turbine is used for small changes of mechanical power, and can be obtained by linearising equations 2, 3 and 4 about an initial operating point to give

$$\frac{d\Delta q}{dt} = -\frac{\Delta h}{T_w}, \Delta q = \frac{\partial q}{\partial c} \Delta c + \frac{\partial q}{\partial h} \Delta h, \Delta P_m = \frac{\partial P_m}{\partial h} \Delta h + \frac{\partial P_m}{\partial q} \Delta q \quad (6)$$

Introducing the Laplace operator  $s$  and eliminating  $\Delta h$  and  $\Delta q$  from the equations gives

$$\frac{\Delta P_m}{\Delta c} = \frac{\left[ \frac{\partial q}{\partial c} \frac{\partial P_m}{\partial q} - T_w s \frac{\partial P_m}{\partial h} \frac{\partial q}{\partial c} \right]}{1 + T_w s \frac{\partial q}{\partial h}} \quad (7)$$

where the partial derivatives are

$$\frac{\partial q}{\partial h} = \frac{1}{2} \frac{c_0}{\sqrt{h_0}}; \quad \frac{\partial q}{\partial c} = \sqrt{h_0}; \quad \frac{\partial P_m}{\partial h} = A_t h_0;$$

$$\frac{\partial P_m}{\partial h} = A_t (q_0 - q_{nl}) \approx A_t q_0 \quad (8)$$

and the suffix "0" indicates an initial value. Substituting into equation 7 gives

$$\frac{\Delta P_m}{\Delta c} = A_t h_0^{3/2} \frac{1 - T_w' s}{1 + \frac{T_w'}{2} s} \quad (9)$$

where  $T_w' = T_w \frac{q_0}{h_0} = \frac{L}{Ag} \frac{Q_0}{H_0}$ . Typically  $T_w'$  is between 0.5 and 5 [s].

This is the classic definition of water starting time but is dependent on the values of the head and the flow rate at the linearization point. It therefore varies with load. If required, the

Some of the authors [2], [3], [5] propose mathematical model modifications using model acceptance measurements data, such is relationships between Mechanical Turbine Power and water flow rate  $P = f(q)$ , gate opening and water flow rate  $c = f(q)$ . Relationship between Mechanical Turbine Power and water flow rate obtained from model acceptance measurements is given in figure 5.

In this paper is proposed another modification of nonlinear hydraulic turbine model which includes relationship between efficiency and gate opening  $\eta = f(c)$ , and combination

constant  $A_t$  can be absorbed into the gate position when it effectively converts the gate opening to per unit turbine power on the generator base. The block diagram of the linearized hydraulic turbine model is shown in figure 3.

Equation 9 describes an interesting and important characteristic of water turbines. For example, suppose that the position of the gate suddenly closed slightly so as to reduce the turbine power output. The flow rate in the penstock cannot change instantaneously so the velocity of the flow through the turbine will initially increase. This increase in water velocity will produce an initial increase in the turbine power until, after a short delay, the flow rate in the penstock has time to reduce when the power will also reduce. This effect is reflected in equation 9 by the minus sign in the numerator. This characteristic is shown in figure 3 where a step increase in the gate position  $\Delta c$  initially produces a rapid drop in power output. As the flow rate in the penstock increases the power output increases.

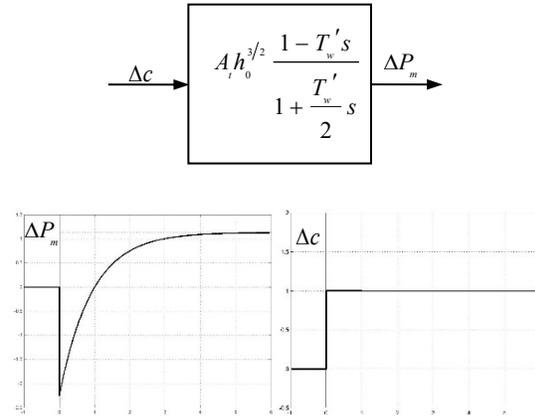


Figure 3. Linear model of hydraulic turbine and its response to a step change in gate position

### III. MODIFICATIONS OF THE MATHEMATICAL MODEL OF HYDRAULIC TURBINE

between  $\eta = f(c)$  and  $P = f(q)$  which are shown on figure 4. The relationship between efficiency and gate opening also obtained from model acceptance measurements is shown on figure 6. On figure 7 is shown comparison between static characteristics of nonlinear, linear and both modifications of nonlinear model with measured static characteristic obtained from model acceptance measurements. It can be concluded that the modification of nonlinear hydraulic turbine model which includes relationship between efficiency and gate opening  $\eta = f(c)$ , have almost identical static characteristic with the measured one, and because of that it will be used for further analysis.

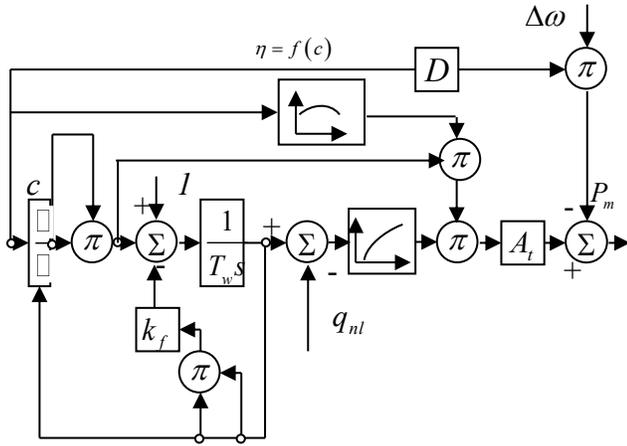


Figure 4. Nonlinear model of hydraulic turbine including efficiency - gate opening and P - q modification

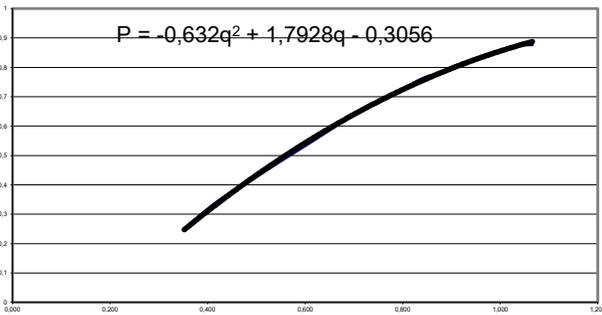


Figure 5. Relationship between Mechanical Turbine Power and water flow rate

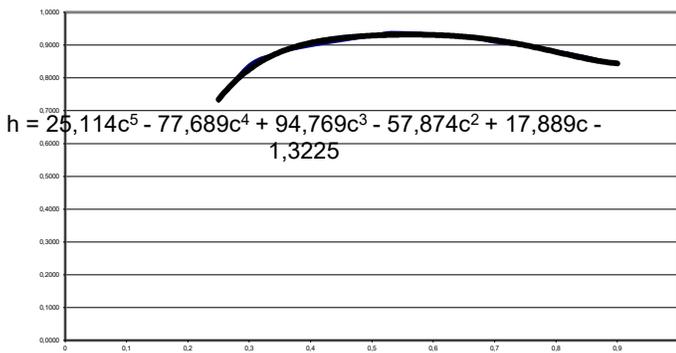


Figure 6. Relationship between efficiency and gate opening

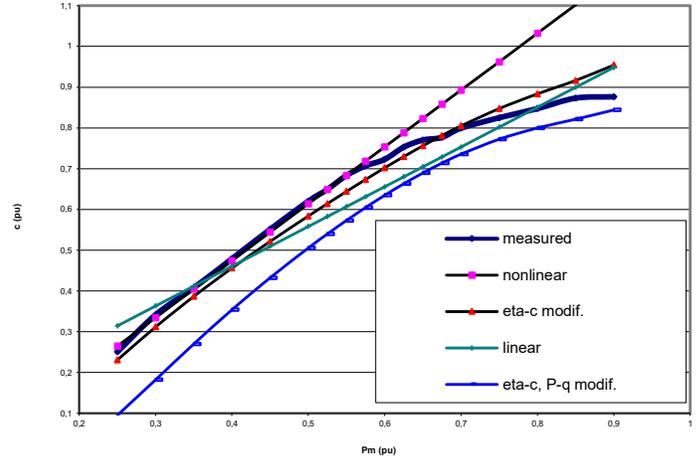


Figure 7. Mechanical Power versus gate opening for change of gate opening from 0.25 to 0.9 pu

In order to improve the accuracy of the results of the simulations of the Francis turbine hydropower model obtained above, a modification of the output mechanical power with an empirical dependence function between the independent variable in the mathematical model as well as the correction function of the output model should be performed. The correction of a prototype of a real turbine plant where the functions used to modify an existing nonlinear block diagram are dependent on the relationship between the efficiency coefficient and the gate opening  $\eta = f(c)$  and suitable coefficient of amplification  $A_t$  which greatly improves the simulation results.

The graphical dependencies between the measured variables and the conductivity of the conducting apparatus are shown in Fig. 8. From the diagram in Fig. 8, we can determine the amplification coefficient  $A_t$  as the coefficient of direction of the tangent to the curve at the positive part of the abscissa. The volume flow losses through the turbine marked with  $q_{nl}$  are obtained for the gate opening when the output power is equal to zero i.e. for the gate opening for which the curve cuts the abscissa.

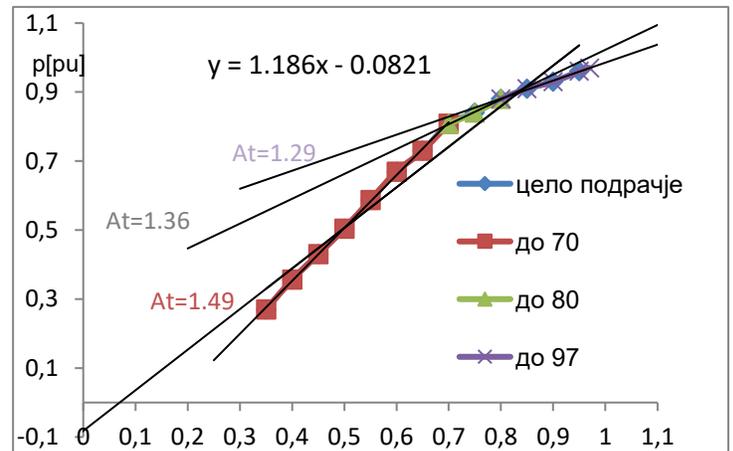


Figure 8. Mechanical Power versus gate opening for and definition of  $A_t$  at  $q_{nl}$  coefficients

IV. SIMULATION PARAMETERS AND DIAGRAMS

The simulations of static and dynamic characteristics of models of hydraulic turbine with governor are made using MATLAB's SIMULINK, with step change of gate position which is input into system, shown on the figure 9.

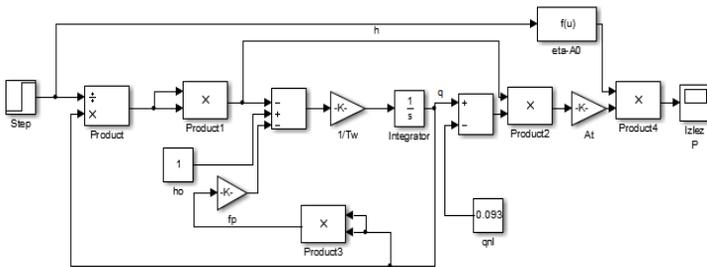


Figure 9. Simulation diagram of the turbine in Simulink

The parameters used in simulations are:

Table 1. Parameters for the simulation in Simulink

Turbine max. Power	20010 [kW]
Turbine nominal Power	18200 [kW]
Max. Head	42,4 [m]
Base Head	40 [m]
Max. Flow rate	53,5 [m <sup>3</sup> /s]
Nominal Flow rate	50 [m <sup>3</sup> /s]
Water starting time $T_w$	0,479 [s]
Penstock head loss	0,000225
Proportionality factor $A_t$	1.186
Proportionality factor $A_t$ for gate opening $c=0,35 - 0,7$	1.490
Proportionality factor $A_t$ for gate opening $c=0,7 - 0,8$	1.360
Proportionality factor $A_t$ for gate opening $c=0,8 - 0,97$	1.290
No load flow $q_{nl}$	0.093 [pu]

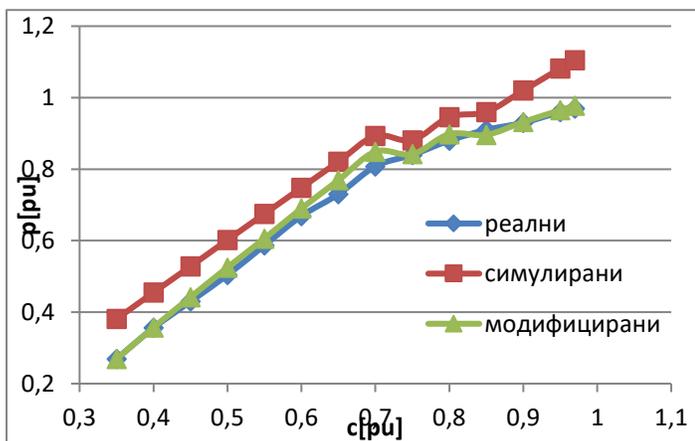


Figure 10. Simulation output diagram for mechanical Power versus gate opening for the modified model.

From simulation of dynamic characteristics of all models it can be concluded that all models are stable in whole range of operating conditions and input changes of network load. In figure 0 is shown that in comparison of the results from figure 7 the output error of the simulated mechanical power is less than

4%, which is not the case for the gate opening bigger than 0,7 pu for all of the models in figure 7. The complete system output of hydraulic turbine and governor tends to keep the reference rotor speed and satisfies the standards for production of electrical power.

## V. CONCLUSION

The advantages of nonlinear mathematical model versus linear mathematical model become apparent when both models are subjected to large excursions in turbine loading, but nonlinear model although good, not always satisfies all demands for specific turbine. Because of that, additional modifications are made, to increase model similarity with real turbine, and to give results close to real measurements on turbine, which can be later used in control tasks, to increase the turbine efficiency. Must be noted [2] that the modifications which are made, can be used only on turbine for which model acceptance tests are made, and they minimize the model generality. The simulation model used in this paper is improving general accuracy of the output results, but it can be designed only after model acceptance tests, because the variable  $A_t$  parameter can be obtained only after experimental measurements.

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