

**Faculty of Engineering  
University of Kragujevac**



**WORKBOOK FOR ENTRANCE EXAM**



Kragujevac, 2024.

The entrance exam for enrollment in the Faculty of Engineering is the first obstacle placed in front of the future students.

When candidates apply for the entrance exam, they will take the mathematics entrance exam.

The test covers the following areas of mathematics:

Algebraic and numerical expressions

Functions, linear functions, equations, inequalities and systems

Quadratic functions, quadratic equations, inequalities and systems

Irrational equations and inequalities

Exponential function, equations and inequalities

Concept of logarithm, logarithmic function, equations and inequalities

Polynomials

Trigonometric functions

Trigonometric equations and inequalities

Complex numbers

Analytical geometry in the plane

Planimetrics

Stereometry

Arithmetic and geometric sequences

To prepare for the entrance exam, you can also use the Collection of test from the entrance exams held in previous years, which is available in this workbook.

**Entrance exam in MATHEMATICS****for enrolment into bachelor academic studies in Electrical Engineering and Computer Science**June 30<sup>th</sup>, 2021

Exam duration is 180 minutes. The test has 15 assignments. A student shall obtain 4 points for each correctly circled answer. Circling the wrong answer, circling more than one answer, as well as leaving the assignment unanswered brings neither positive nor negative points. The use of calculator is not allowed.

1. If  $a = \frac{\sqrt{2}-1}{\sqrt{2}+1}$  and  $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$  then the value of the expression  $(a + a^{-1} + b + b^{-1})^{\frac{1}{2}}$  equals:

- A) 2      B) 3      C) 4      D)  $2\sqrt{2}$       E)  $2\sqrt{5}$       F) it is not stated

2. If function  $f: R \rightarrow R$  is given with  $f(x) = x^2 - 2x + 5$ , then  $-f(2-x)$  equals:

- A)  $2x - f(x)$       B)  $x - f(x)$       C)  $-f(x)$       D)  $2 + f(x)$       E)  $-f(x) + 2$       F) it is not stated

3. The sum of all values of the real parameter  $p$  for which the sum of the squares of the roots of quadratic equation  $2x^2 - px - 2p + 3 = 0$  equals 2 is:

- A) -8      B) -6      C) 2      D) 3      E) 11      F) it is not stated

4. If  $z = \frac{2-i}{-1-i}$ , where  $i^2 = -1$ , then the expression  $\operatorname{Re}(z) + (\operatorname{Im}(y))^2$  has the value:

- A) -2      B)  $-\frac{1}{2}$       C) 1      D)  $\frac{3}{2}$       E)  $\frac{7}{4}$       F) it is not stated

5. A square is inscribed in an isosceles right-angled triangle so that two of its vertices lie on the hypotenuse and two other vertices lie on each leg of the triangle. If the length of the leg of a right triangle is  $12\sqrt{2}$ , then the area of the square is (in  $\text{cm}^2$ ):

- A) 25      B) 36      C) 49      D) 64      E) 81      F) it is not stated

6. If the remainder of the division of polynomial  $P(x) = x^3 + 9x^2 + ax + b$  by binomial  $x + 1$  equals 4, and the remainder of the division by binomial  $x - 1$  equals 24, then the sum of numbers  $a + b$  equals:

- A) -14      B) -12      C) 0      D) 14      E) 16      F) it is not stated

7. The sum of all real solutions of equation  $|4x^2 - 4x - 3| = 2x + 1$  is:

- A)  $\frac{1}{2}$       B) 3      C)  $\frac{3}{2}$       D)  $\frac{7}{2}$       E)  $\frac{5}{2}$       F) it is not stated

8. If the number  $\frac{m}{n}$  is a solution of equation  $9^x - 2^{x+\frac{1}{2}} = 2^{x+\frac{7}{2}} - 3^{2x-1}$  where  $m$  and  $n$  are relatively prime natural numbers, then  $m+n$  equals:

- A) 3      B) 5      C) 7      D) 9      E) 11      F) it is not stated

9. The solution of equation  $\sqrt{x^2 - 5x + 4} \leq x - 3$  is the set:

- A)  $[4, 5]$       B)  $[3, 5]$       C)  $(-\infty, -1] \cup [4, 5]$       D)  $(-\infty, -1] \cup [4, \infty)$       E)  $[3, \infty)$   
 F) it is not stated

10. If the length of the height of a regular tetrahedron is equal to  $\sqrt{3}$ , then its area is equal to:

- A)  $3\sqrt{3}$       B)  $\frac{9\sqrt{3}}{4}$       C)  $\frac{9\sqrt{3}}{2}$       D)  $\frac{9\sqrt{3}}{\sqrt{2}}$       E)  $\frac{9\sqrt{6}}{4}$       F) it is not stated

11. In the expansion of the binomial power  $\left(\frac{1}{2} + \sqrt[3]{x}\right)^{10}$  the coefficient with  $x^2$  is:

- A)  $\frac{105}{8}$       B)  $\frac{105}{32}$       C)  $\frac{1}{32}$       D)  $\frac{1}{16}$       E) 210      F) it is not stated

12. Geometric sequence  $(a_n)$  is such that  $a_1 + a_2 + a_3 = 26$  and  $a_1 + a_3 = 20$ . If  $a_1 < 10$  then  $a_5$  equals:

- A) 142      B) 152      C) 162      D) 172      E) 192      F) it is not stated

13. How many five-digit numbers with different digits are there, divisible by 5, whose digits are from the set of 0,1,2,3,5?

- A) 40      B) 42      C) 44      D) 46      E) 48      F) it is not stated

14. Equation of the circle which is concentric to the circle  $x^2 + y^2 + 6x + 2y + 5 = 0$  and passes through the point  $M(1, -4)$  is:

- A)  $(x-3)^2 + (y-1)^2 = 5$       B)  $(x+3)^2 + (y+1)^2 = 5$       C)  $(x+3)^2 + (y+1)^2 = 16$   
 D)  $(x+3)^2 + (y+1)^2 = 25$       E)  $(x-3)^2 + (y-1)^2 = 25$       F) it is not stated

15. The number of solutions of equation  $\sin x + \frac{1}{\sqrt{3}} \sin 2x = 0$  on the interval  $[0, 2\pi]$  is:

- A) 0      B) 1      C) 2      D) 4      E) 5      F) it is not stated

## Entrance exam in MATHEMATICS

for enrolment into bachelor academic studies in Electrical Engineering and Computer Science

June 29<sup>th</sup>, 2022

Exam duration is 180 minutes. The test has 15 assignments. A student shall obtain 4 points for each correctly circled answer. Circling the wrong answer, circling more than one answer, as well as leaving the assignment unanswered brings neither positive nor negative points. The use of calculator is not allowed.

1. Real part of the complex number  $z = (-1 + 5i) \cdot \left(2 - \frac{3+i}{2+i}\right)$  is:

- A) 8      B) -8      C) 1      D)  $\frac{10}{3}$       E)  $\frac{1}{8}$

2. Solution set of equation  $\sqrt{2x+14} - \sqrt{x-7} = \sqrt{x+5}$  is:

- A) {11}      B) {-9,11}      C) {-9}      D) {-11,9}      E)  $\emptyset$

3. The sum of all values of the parameter  $m \in R$  so that the solutions of equation

$(m+2)x^2 - 2(m+1)x + m = 0$  satisfy the condition  $x_1^2 + x_2^2 = \frac{10}{9}$ , is:

- A)  $\frac{3}{2}$       B)  $\frac{1}{2}$       C) 1      D) -1      E)  $-\frac{1}{2}$

4. The coefficient of the second term in the expansion of binomial  $\left(\frac{x}{\sqrt{y}} + \frac{\sqrt{y}}{x}\right)^n$  has a ratio towards the coefficient of the third term of 2:11. The term with  $x^4$  is:

- A)  $495x^4y^2$       B)  $462x^4y^2$       C)  $495x^4y^{-2}$       D)  $792x^2y^{-1}$       E) such term does not exist

5. The number of real solutions of equation  $\log_{11}(x+2) + \log_{\frac{1}{11}}(2x-3) = 0$  is:

- A) 0      B) 1      C) 2      D) 3      E) 4

6. The number of solutions of trigonometric equation  $2\sin 3x + \sqrt{2} = 0$  on the interval  $(0, 4\pi)$  is:

- A) 2      B) 12      C) 4      D) 8      E) 10

7. A right trapezoid with bases  $a = 9 \text{ cm}$  and  $b = 6 \text{ cm}$  is circumscribed about a circle. The area of this trapezoid is:

A)  $72 \text{ cm}^2$     B)  $270 \text{ cm}^2$     C)  $54 \text{ cm}^2$     D)  $\frac{216}{25} \text{ cm}^2$     E)  $108 \text{ cm}^2$

8. If function  $f : \mathbb{R} \rightarrow \mathbb{R}$  is given with  $f(x) = \sqrt[3]{x^3} + 2|x| - 7x^2$ , than  $f(1) + f(-1)$  is:

A) -2    B) 2    C) -14    D) 18    E) -10

9. Three numbers whose sum is 26 are consecutive members of a geometric sequence. If 1, 6 and 3 are added to those numbers respectively, three numbers are obtained that make up an arithmetic sequence. The product of those numbers is:

A) 432    B) 216    C) 108    D) 324    E) none of the previous answers

10. How many ways are there to distribute numbers 1, 2, ..., 2000 so that no two adjacent numbers have an even sum?

A)  $(1000!)^2$     B)  $2 \cdot 1000!$     C)  $(2 \cdot 1000!)^2$     D)  $2 \cdot (1000!)^2$     E)  $2000 \cdot 1000!$

11. In a straight cup of height  $H=6$  and base radius  $r=4$ , a cube is inscribed with edge  $a$ , one side of which lies on the base of the cup. The edge of the cube is:

A)  $\frac{24}{4+3\sqrt{2}}$     B)  $\frac{12}{5}$     C)  $\frac{24}{7}$     D)  $4+3\sqrt{2}$     E) 12

12. The solution of the system of inequalities  $x^2 - x - 2 < 0$  and  $-x^2 + 4x - 3 < 0$  is:

A)  $(-\infty, -1)$     B) (1,2)    C) (1,3)    D) (-1,1)    E) (-1,3)

13. The solution of the given equation  $4^x - 3^{x+\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x+1}$  belongs to the interval:

A) (0,1)    B) (-1,0]    C) (2,3)    D) [3,4]    E) (4,+∞)

14. If polynomial  $P(x) = x^4 + 6x^3 - 8x^2 + ax + b$  is divisible by polynomial  $x^2 - 3x + 2$  than  $b - a$  equals:

A) -76    B) -67    C) 1    D) 67    E) 76

15. The equation of the mutual chord of circles  $x^2 + y^2 = 10$  and  $x^2 + y^2 - 6x - 6y + 2 = 0$  is:

A)  $2x + 3y - 2 = 0$     B)  $x + y + 2 = 0$     C)  $x - y - 2 = 0$     D)  $x - y + 2 = 0$     E)  $x + y - 2 = 0$

Entrance exam in MATHEMATICS

for enrolment into bachelor academic studies in Electrical Engineering and Computer Science

June 28th, 2023

Exam duration is 180 minutes. The test has 15 assignments. A student shall obtain 4 points for each correctly circled answer. Circling the wrong answer, circling more than one answer, as well as leaving the assignment unanswered brings neither positive nor negative points. The use of calculator is not allowed.

1. How many whole number solutions exist in relation to inequality  $\frac{x^2 - 2x + 2}{x + 1} \geq 0$   
 a) 0                      b) 2                      c) 5                      d) infinitely many                      e) 3
2. The sum of all the solutions of equation  $2 \cos^2 x + 3 \sin x = 0$  on the interval  $(0, 2\pi)$  is:  
 a)  $\frac{7\pi}{6}$                       b)  $\frac{11\pi}{6}$                       c)  $3\pi$                       d)  $2\pi$                       e) 0
3. If  $f\left(\sqrt{\frac{x-2}{x+1}}\right) = x$ , then  $f(2)$  equals:  
 a) -2                      b) -1                      c) 0                      d) 1                      e) 2
4. Polynomial  $P(x) = x^4 + ax^2 + 4x + b$  is divisible by polynomial  $Q(x) = x^2 + 2x + 1$ . Then  $4a + b$  equals:  
 a) 6                      b) 5                      c) 4                      d) 3                      e) 2
5. Real part of the solution of equation  $z + |z + 2i| = 2i + 1$ , where  $i^2 = -1$  is:  
 a) 5                      b)  $-\frac{15}{2}$                       c) 2                      d)  $\frac{17}{2}$                       e)  $-\frac{17}{2}$
6. The sum of all rational solutions of equation  $25^{2x-x^2+1} + 9^{2x-x^2+1} = 34 \cdot 15^{2x-x^2}$  is:  
 a) 2                      b) 3                      c) 6                      d) 0                      e) 4
7. The product of solutions of equation  $\log 2 + \log(4^{x-2} + 9) = 1 + \log(2^{x-2} + 1)$  is:  
 a) 32                      b) 16                      c) 8                      d) 2                      e) 20
8. The solutions of equation  $\sqrt{10+x} - \sqrt{10-x} = \sqrt{2x-8}$  belong to the interval:  
 a)  $(-\infty, 2)$                       b)  $[2, 6)$                       c)  $[6, 10)$                       d)  $[10, 20)$                       e)  $[20, +\infty)$
9. Circle contains points  $A(-1, 2)$  and  $B(3, 4)$ , and the center of the circle belongs to infinite line  $y = x - 7$ . The length of the circle radius is:  
 a)  $3\sqrt{5}$                       b)  $5\sqrt{2}$                       c) 7                      d)  $4\sqrt{3}$                       e)  $2\sqrt{13}$
10. Arithmetic sequence is given where  $2a_2 - a_4 + a_5 = 19$  and  $a_6 + a_7 = 43$ . The sum  $a_{22} + a_{23} + \dots + a_{30}$  equals:  
 a) 720                      b) 800                      c) 815                      d) 652                      e) 755

11. The sum of the binomial coefficients of the last three terms in the expanded form of the binomial  $(x^2 + \frac{1}{x})^n$  equals 46. The term that does not contain x is the:

- a) fifth      b) sixth      c) seventh      d) eighth      e) such term does not exist

12. The number of ways to form a five member committee out of two mathematicians and eight engineers in which at least one member shall be a mathematician is:

- a) 56      b) 70      c) 140      d) 196      e) 248

13. If the volume of a regular tetrahedron is  $144\sqrt{2}$ , then the length of the sphere radius inscribed in that tetrahedron is:

- a)  $2\sqrt{6}$       b)  $\sqrt{6}$       c)  $\frac{3}{2}\sqrt{6}$       d)  $\frac{2}{3}\sqrt{6}$       e)  $2\sqrt{3}$

14. The value of the expression  $\left(\frac{\sqrt{3}+2}{2-\sqrt{3}} + \frac{\sqrt{3}-2}{2+\sqrt{3}}\right)^{-2}$  is equal to:

- a)  $4\sqrt{3}$       b) 14      c)  $\frac{3}{4}\sqrt{6}$       d)  $\frac{1}{192}$       e)  $\frac{1}{108}$

15. The length of the side AB of the triangle ABC is 4 cm, and its corresponding height is  $\sqrt{3}$  cm. If the angle at the vertex A equals  $60^\circ$ , then the length of the side BC equals:

- a)  $3$  cm      b)  $3\sqrt{3}$  cm      c)  $2\sqrt{3}$  cm      d) 4 cm      e)  $3\sqrt{2}$  cm